

Surprises in Bose-Einstein correlations

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February 7, 2008

Abstract

Four experimental results, which seem to contradict the established ideas about the Bose-Einstein correlations in multiple particle production processes, are briefly presented and discussed.

1 INTRODUCTION

The study of Bose-Einstein correlations (further quoted BEC) in multiple particle production processes is an important source of information about the interaction region, i.e. about the region, where the hadrons are produced. One would like to know the size and shape of this region as well as its orientation with respect to the momentum of the incident particle and to the impact parameter vector. For the matter inside one would like to know: its flows, its equation of state and its phase transitions. Much information has already been obtained (cf. the reviews [1], [2], [3], [4] and references quoted there), its reliability, however, depends on the correctness of our interpretation of the observed BEC.

Four surprising experimental results concerning BEC will be here briefly presented and discussed. By surprising we do not just mean that they disagree with some specific model, but that they seem to contradict the basic physical pictures used to build most of the currently popular models. Perhaps in the future some trivial explanations will be found, but if not, these observations may lead to a reinterpretation of the BEC data and consequently to changes in our conclusions concerning the interaction region. Thus the questions raised are interesting and potentially important.

*Supported in part by the KBN grant 2P03B 093 22

2 No inter- W BEC at LEP

At LEP2 the reactions

$$e^+e^- \rightarrow W^+W^- \rightarrow q\bar{q}l\nu \quad (1)$$

$$e^+e^- \rightarrow W^+W^- \rightarrow q\bar{q}q\bar{q} \quad (2)$$

have been observed. In the first, further denoted W , one of the W -bosons decays into hadrons and the other one into leptons. In the second, further denoted WW , both W -s decay into hadrons. Assuming that in reaction WW there are no inter- W correlations, it is a standard exercise in the calculus of probabilities to express any two-particle distribution for reaction WW in terms of the corresponding single-particle and two-particle distributions for reaction W . One finds [5]

$$\rho_2^{(WW)}(p_1, p_2) = 2\rho_2^{(W)}(p_1, p_2) + 2\rho_1^{(W)}(p_1)\rho_1^{(W)}(p_2). \quad (3)$$

The surprise is that this formula works – there is no evidence for inter- W correlations [6],[7],[8],[9],[10].

There is a number of known factors, which reduce the inter- W BEC (cf. e.g. [11]). The two W -s decay in different places and have non-zero relative velocity. The partons from one W can form hadrons with the partons from the other. This effect – known as color reconnection – shortens the chain of interactions leading to the final hadrons and consequently reduces the multiplicity of the final particles. Since BEC increase the particle multiplicity, the two effects tend to cancel. Nevertheless, the observed reduction of the inter- W BEC seems to be much stronger than what can be easily understood.

One can try various explanations of this fact. The belief that BEC for identical bosons must lead to attraction in momentum space is based on a restricted class of models, where the two-particle density matrix in the momentum representation is a symmetrized product of single particle density matrices [12],[13]. This does not have to be the case. For instance the amplitude

$$\psi(p_1, p_2) = \text{Sign}[(p_1 - p_2)(x_1 - x_2)] \times (e^{ip_1x_1 + ip_2x_2} - e^{ip_2x_1 + ip_1x_2}) \quad (4)$$

has the correct symmetry, but corresponds to a different density matrix and leads to repulsion in momentum space instead of the attraction. Perhaps one should look for more general density matrices than those currently used. Häkkinen and Ringnér [14] suggested that the absence of the inter- W correlations is natural in the framework of string models and postulated it. This

point of view was supported by Bo Andersson (cf. e.g. [15]). Up to now, however, no convincing motivation for it has been published.

3 The Bertsch-Pratt radii

The next three surprising results have come from the study of heavy ion collisions at RHIC. In order to present them it is convenient to introduce the Bertsch-Pratt coordinate frame [16], [17], [18]. This is a Cartesian frame with its axis L – for longitudinal – along the event axis. For heavy ion collisions the event axis is along the direction of the momentum difference of the two nuclei in the centre of mass system or equivalently in the laboratory system. The other two axes are in the plane perpendicular to L and are defined separately for each pair of particles. Denoting the four-momenta of the two particles forming the pair by p_1 and p_2 let us define two more four-vectors

$$K = \frac{1}{2}(p_1 + p_2); \quad q = p_1 - p_2. \quad (5)$$

The second axis of the Bertsch-Pratt frame – denoted *out* – is parallel to the transverse (with respect to L) component of \mathbf{K} . The third axis – denoted *side* – is perpendicular to the L and *out* axes, or equivalently to the L -axis and to the \mathbf{K} vector.

The correlation function measured in experiment, after a number of corrections which will not be discussed here, is parameterized as

$$C(K, q) = 1 + \lambda e^{-R_L^2(K_T)q_L^2 - R_o^2(K_T)q_o^2 - R_s^2(K_T)q_s^2}. \quad (6)$$

The coefficient λ is a constant and $R_i(K_T)$ are the three Bertsch-Pratt radii. The following three assumptions had been accepted.

- The momenta \mathbf{p}_i are measured in the local co-moving system (LCMS) obtained from the laboratory (or cms.) system by a Lorentz transformation along the L direction such that in the LCMS: $K_L = 0$.
- The distribution of momenta \mathbf{p}_i is invariant with respect to boosts along the L axis. This is known as boost invariance [19].
- The distribution of pairs of momenta $\mathbf{p}_1, \mathbf{p}_2$ does not change under rotations around the L axis. This means either that the collisions are central, or that the data is averaged over the angle between \mathbf{K}_T and the impact parameter vector \mathbf{b} .

Under these assumptions vector \mathbf{K} in the arguments of the $R_i(K)$ can be replaced by the length of its transverse component $K_T = |\mathbf{K}_T|$. The dependence on the time component of K , which is allowed, is not written explicitly. Let us consider now the three surprising results referred to collectively as the RHIC (BEC) puzzle.

4 Approximate equality $R_s(K_T) \approx R_o(K_T)$

At $\mathbf{K}_T = 0$ each direction perpendicular to L can be considered parallel as well as perpendicular to \mathbf{K}_T and consequently

$$R_o(0) = R_s(0). \quad (7)$$

The relation of these parameters to the space distribution of the sources of pions is

$$R_o^2(0) = \langle \tilde{x}_{out}^2 \rangle_0, \quad (8)$$

$$R_s^2(0) = \langle \tilde{x}_{side}^2 \rangle_0, \quad (9)$$

where $\tilde{x} = x - \bar{x}$ is the deviation of the position four-vector of the source from a fixed space time point \bar{x} corresponding to the average position of the source in space-time. The averaging is over all the pairs of particles, which have $K_T = 0$ and are of the type considered, say π^+ . We use the notation $\tilde{x} = \{\tilde{x}_L, \tilde{x}_{out}, \tilde{x}_{side}, \tilde{t}\}$. For reasons discussed below models suggest that the ratio $R_s(K_T)/R_o(K_T)$ should increase with increasing K_T [20], [21]. It came, therefore, as a surprise that this ratio remains approximately equal one, when K_T increases from zero [20], [21], [22]. What is more, subsequent data indicate that the ratio $R_o(K_T)/R_s(K_T)$ decreases below one with increasing K_T [20], [23], [24] as well as with decreasing impact parameter of the collision [23].

The transition from $K_T = 0$ to $K_T \neq 0$ corresponds to a Lorentz transformation with velocity

$$\beta_T = \frac{K_T}{K_0} \quad (10)$$

along the direction of \mathbf{K}_T . Therefore, since the dimensions orthogonal to the direction of the boost remain unchanged, one should expect

$$R_s^2(K_T) = \langle \tilde{x}_{side}^2 \rangle_0, \quad (11)$$

while the dimension perpendicular to the boost gets transformed according to the usual rules and one should get

$$R_o^2(K_T) = \langle \tilde{x}_{out}^2 \rangle_0 - 2\beta_T \langle \tilde{x}_{out} \tilde{t} \rangle_0 + \beta_T^2 \langle \tilde{t}^2 \rangle_0. \quad (12)$$

The standard recommendation had been to identify the spatial radius in the transverse direction with R_s , to neglect the correlation term $\langle \tilde{x}_{out} \tilde{t} \rangle_0$ and to calculate the time span of the production process from the formula (cf. e.g. [20]):

$$\tau_0^2 \equiv \langle \tilde{t}^2 \rangle \approx \langle \beta_T^{-2} \rangle (R_o^2 - R_s^2). \quad (13)$$

This, however, becomes incredible when $R_o \approx R_s$ and absurd when $R_o < R_s$.

An obvious improvement is to take into account the correlation term. The hydrodynamic models, however, predict

$$\langle \tilde{x}_o \tilde{t} \rangle < 0, \quad (14)$$

which makes the situation even worse. On the other hand, the result of the hydrodynamic model has a clear physical interpretation and rejecting it is a serious decision. The picture behind it is that in the (expanding) cylindrical interaction volume hadronization begins at the surface and progresses inwards, so that particles produced at small \tilde{x}_{out} are produced late. The opposite prediction

$$\langle \tilde{x}_o \tilde{t} \rangle > 0 \quad (15)$$

occurs in the so-called microscopic models [25], [26] where the expansion according to Euler's hydrodynamics is replaced by an expansion according to some approximation to Boltzmann's equation. The physical interpretation of this result is not quite clear yet, but at present the predicted correlation is too weak to explain the observed decrease of the ratio $R_o(K_T)/R_s(K_T)$. The description of some more ideas on how to solve this part of the puzzle can be found in the review paper [27]. All these approaches, however, seem to have serious problems.

5 Approximate equality $\frac{\partial R_s}{\partial K_T} \approx \frac{\partial R_o}{\partial K_T}$

Experimentally the transverse momentum dependence of R_s and of R_o is similar – both drop with increasing K_T . Studies of the ratio $R_s(K_T)/R_o(K_T)$ indicate that R_s drops somewhat faster than R_o . Since data does not start at $K_T = 0$, this does not necessarily mean that $R_o(K_T) < R_s(K_T)$. There

could be a rise of $R_o(K_T)/R_s(K_T)$ in the small K_T region. Nevertheless, these observations contradict the following simple argument consistent with most models.

Let us consider the case, when K_T is large. Then \mathbf{K}_T should be strongly correlated with the transverse velocity \mathbf{v}_T of the corresponding element of the expanding gas or liquid. It should not be a bad approximation to assume that roughly the directions of \mathbf{K}_T and \mathbf{v}_T coincide. Thus, the *out* components of the particle momenta are generated mostly by the collective velocity \mathbf{v}_T and less by the velocities of the particles in the rest frame of the element, which should be roughly isotropic, characterized by some temperature T . The side components of the momenta, on the other hand, result mostly from the internal motion – thermal in thermodynamic models. This picture implies that $R_o(K_T)$ is likely to change with K_T , while little or no change is expected for $R_s(K_T)$. The expectation

$$\frac{\partial R_s}{\partial K_T} \approx 0, \quad (16)$$

which as we now know contradicts experiment, was an important point in Bertsch's argument in favor of the $L, out, side$ coordinate frame [18]. It is amazingly robust and holds not only in hydrodynamic models, where local temperature is a natural concept, but also in microscopic models [23].

6 R_L smaller than expected

It is well known that the parameter R_L measures the size of the "homogeneity region" and not the total size of the interaction region in the L direction. The reason is that only pairs of identical particles with similar momenta contribute to BEC and, because of the strong $\mathbf{p}-x$ correlations at production, the particles with similar momenta cannot be produced very far from each other. Models typically give

$$R_L = \tau_0 \sqrt{\frac{T}{M_T}} f\left(\frac{T}{M_T}, \frac{K_T}{K_0}\right), \quad (17)$$

where τ_0 is the life time of the interaction region (not to be confused with the much shorter time interval, where hadronization takes place), T and M_T are respectively the temperature and the transverse mass of the pair of particles and f is a slowly varying function. The simplest choice – $f = \text{Const}$ – has been made by Makhlin and Sinyukov [28]. The formula

$$R_L = \frac{A}{\sqrt{M_T}}, \quad (18)$$

where A is a constant, has been successfully used to fit the data [21]. The problem is that the experimental values of the constant A , which rises from 2.19 ± 0.05 fm GeV^{1/2} at AGS energies ($\sqrt{s} = 4.1/4.9$ GeV/nucleon) to 3.32 ± 0.03 fm GeV^{1/2} at RHIC energies ($\sqrt{s} = (130 - 200)$ GeV/nucleon) [21], are much smaller than expected [27].

The difficulty can be overcome in a number of ways. One can choose a very small value of τ_0 , which corresponds to expansion with ultrasonic speed [27], or assume early chemical freeze-out, i.e. no chemical equilibrium among the final particles, [29]. The trouble is that these modifications do not solve the problem with R_s and spoil the good agreement with experiment for the single particle distributions [27]. Thus, this problem, though perhaps less striking than the other two parts of the RHIC puzzle, remains puzzling.

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